HEAT TRANSFER TO A FLAT PLATE IN A FULLY-DEVELOPED TURBULENT BOUNDARY LAYER

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The authors examine the results of experimental investigation of local and average heat transfer over a flat plate, with the entire plate heated, and with the initial section heated, in the Reynolds number range $2 \cdot 10^2$ to $5 \cdot 10^5$.

Heat transfer to a flat plate in a fully-developed turbulent boundary layer includes two special cases with different boundary conditions. The first case occurs when the heat transfer begins immediately at the leading edge and, therefore, the thermal and dynamic boundary layers form simultaneously (complete heating). With these boundary conditions the Reynolds number in the heat transfer equations is more than 10^5, as a rule, since fully-developed flow exists only outside the dynamic initial section ($X_1 > L_{dyn}$). In the second case the heat transfer does not begin at the leading edge, but further downstream. If the extent of the isothermal initial section exceeds the hydrodynamic stabilization length ($X_0 > L_{dyn}$), then the heated surface lies entirely in the region of developed boundary layer, and the quantity Re_{X_1} in the heat transfer equations can take any value, from zero upwards. These boundary conditions are typical for the thermal initial section.

Heat transfer to a completely heated plate has received detailed experimental study. Correlations have been given in [1, 2]. The thermal initial section, particularly for $Re_{X_1} < 10^5$, has not been investigated fully. Also, the results are contradictory. In [1, 3-6] the heat transfer coefficient in the thermal entrance section is given by the equations for complete heating, using the coordinate X_1 as the governing dimension. However, the experimental data published in [7-16] indicate that the total heating equations overestimate the values of α_k by 10-30% in the thermal entrance section.

A discussion is given below of results of an experimental investigation of local and average heat transfer to a plate washed by a parallel stream of air and with heating over the complete plate and in the thermal initial section.

Local Heat Transfer. Measurements of local heat transfer coefficients were made by a steady-state method with t_w = const on a plate of thickness δ = 8 mm and length L = 280 mm, with a sharp leading edge of angle 90°. A description of the wind tunnel, which has a square working section of size 280×280 mm^2, has been given in [17, 18]. The construction of the plate is shown in Fig. 1. It consists of three water-cooled steel sections 1 with polished working surface, fastened to a common base plate 2 by means of clamps 3. Water enters the internal cavity of each section through the inlet tube 4, and, after passing around a baffle, leaves through the sleeve. The thermocouples 5 measure the initial and final water temperatures. The plate occupies 3% of the wind tunnel cross section.

Five "isolated" heat flux gages 6 and thermocouples 7 [19] are flush-mounted in the surface of the measuring section. The plate is suspended in the center of the wind tunnel, held by the mounting bracket 8 in the ceiling 9. Longitudinal and transverse movements are precluded by means of the rod 10. Interchange of the sections and rotation of the plate through 180° allows the measurement of α_k along the length of the plate at a pitch of 12-15 mm.

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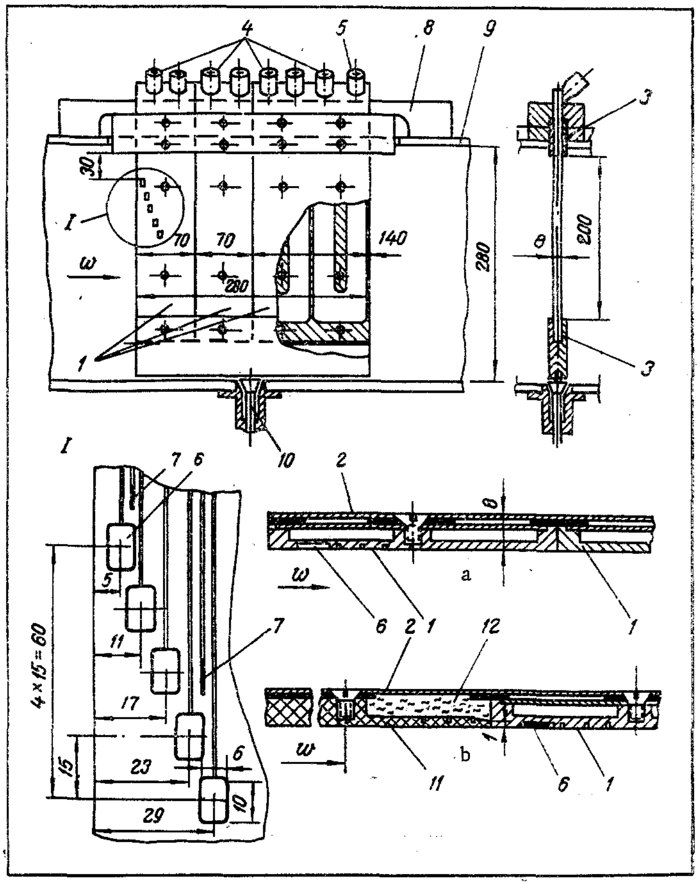

Fig. 1. Construction of the plate: a) assembly of the fully-heated plate; b) assembly of the plate with heated initial section; 1) water-cooled sections; 2) base plate; 3) fittings; 4) tube for cooling water supply; 5) thermocouples for measurement of water temperature; 6) heat flux gages; 7) thermocouples for measurement of wall temperature; 8) bracket; 9) wind tunnel working section ceiling; 10) support rod; 11) plastic textolite strip; 12) loose wadding.

During the tests with the isothermal initial section, one of the two water-cooled sections was replaced by textolite strips of the same size (X_0 = 140 mm or X_0 = 210 mm). To minimize thermal contact between the heated and unheated surfaces of the plate, the thickness of the textolite strip at the interface was reduced to 1 mm, as shown in Fig. 1.

The local heat transfer coefficient at the center of the heat flux gage is given by

$$\alpha_c = \frac{kE}{t_{pot} - t_w} - \alpha_r, \ \mathrm{W/m^2 \cdot deg.} \tag{1}$$

The air temperature in the tests was t_{pot} = 50–55°C, the wall temperature was t_w = 12–18°C, the coefficient of radiative heat transfer was α_r = 2.0–2.5 W/m^2·deg, the air speed was W = 1.5–35.0 m/sec, the turbulence level of the incident flow was $\varepsilon = 2.2 \pm 0.2\%$, and the gage constants fell in the range k = 490–580 W /m^2·μV.* The water flow rate in each section was controlled according to the temperature increment, which was maintained to be 1°C.

The test data obtained with the completely heated plate in the range $Re_X = (30\text{–}500) \cdot 10^3$ are shown in Fig. 2a. In this series of experiments the measurements were made at distances from the leading edge of more than 140 mm, since the flow stabilized at lengths X = (130–140) mm [18], following separation and reattachment at the leading edge. Curve 1 shows the Mikheev formula, Eq. (2a) of [1], which for air

*The construction and calibration of the heat flux gages was performed at the Institute of Thermophysics of the Academy of Sciences of the Ukrainian SSR.

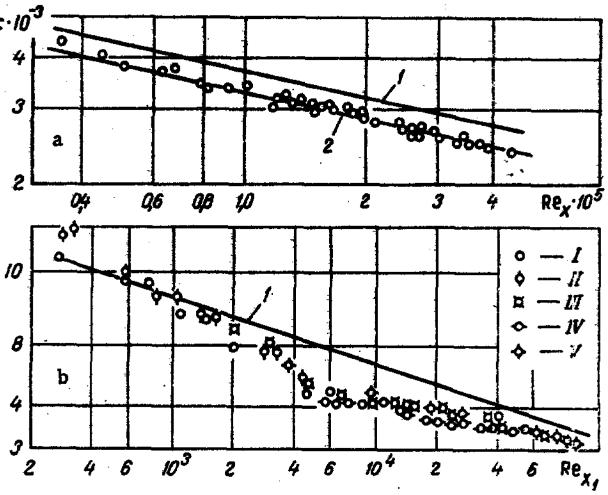

Fig. 2. Results of investigation of local heat transfer to the plate: a) with total complete heating for $X > L_{dyn}$ [1) Eq. (2); 2) Eq. (3)]; b) in the thermal initial section with $X_0 > L_{dyn}$ Eq. (3b); X_0/X: I) 0.98; II) 0.96; III) 0.92; IV) 0.84; V) 0.78].

($Pr \approx 0.71$)* takes the form (2b):

$$Nu_X = 0.0296\, Re_X^{0.8} Pr^{0.43} (Pr_{pot}/Pr_w)^{0.25}, \tag{2a}$$

$$St = Nu_X/Re_X Pr = 0.036\, Re_X^{-0.2}. \tag{2b}$$

Curve 2, which represents the experimental points with a scatter of ±10% for $Re_X > 70 \cdot 10^3$, was calculated from Eq. (3a) of Reynolds et al. [12], which, for air with $Pr \approx 0.71$ and $T_{pot}/T_w < 1.1$, takes the form (3b):

$$St = 0.0296\, Pr_X^{-0.4} Re_X^{-0.2} (T_w/T_{pot})^{-0.4}, \tag{3a}$$

$$St = 0.0325\, Re_X^{-0.2}. \tag{3b}$$

Equation (3a) for the present values of Re_X agrees with the Karman analogy and the relation for the friction factor derived from the velocity distribution in a dynamic boundary layer with a one-seventh power law [12, 20]:

$$c_f = 0.0296\, Re_X^{-0.2}. \tag{4}$$

As can be seen in Fig. 2, Eq. (2b) leads to an overestimate of local heat transfer coefficients by roughly 10%.

The results of investigation of heat transfer on the thermal initial section of the plate for $Re_{X_1} = (0.25\text{–}100) \cdot 10^3$, $X_0/X = (0.75\text{–}0.98)$, and $Re_X > 40 \cdot 10^3$ are shown in Fig. 2b. The experimental points are layered according to the parameter X_0/X. For $Re_{X_1} < 10^4$ the divergence of the experimental points from values calculated according to Eq. (3b) for the Stanton number are particularly noticeable (up to 40%) in the immediate vicinity of the beginning of heating. For $Re_{X_1} > 10^4$ and fixed values of X_0/X the location of the experimental points follows the slope $n = -0.2$. For $Re_{X_1} < 10^4$ this tendency is arrested: the slope decreases monotonically, approaching the value $n = -0.4$.

In [7-16] the experimental local heat transfer data in the thermal initial section were corrected by introducing a factor into the total heating formula:

$$St_0 = St\, \varphi(X_0/X). \tag{5}$$

This correction is based on a correlation of Seban, published first in [13], and then confirmed experimentally and theoretically in [7, 12, 15, 21-23]. With Reynolds number referenced to the heated length the Seban correlation takes the following form:

*The physical constants in the nondimensional numbers St, Re, and Pr here and below are evaluated at flow temperature.

$$\varphi(X_0/X) = \left[1 - \left(\frac{X_0}{X}\right)^{0,9}\right]^{-\frac{1}{9}} \left[1 - \frac{X_0}{X}\right]^{0,2}. \quad (6)$$

Equation (6) and the result of [7, 12, 22] pertain to the region $Re_{X_1} > 10^5$, where the temperature distribution and thermal boundary layer, like the velocity distribution, follows the one-seventh power law. For reduced value of Re_{X_1} the thickness of the thermal boundary layer becomes smaller relative to that of the dynamic boundary layer. For $X_1 \to 0$ the thermal boundary layer is completely contained in the dynamic laminar sublayer, where the velocity and temperature distributions are practically linear.

A theoretical analysis of the heat transfer to a flat plate with a fully-developed turbulent boundary layer and small values of X_1 was given by Kestin and Persen [24]. For Pr = 0.71 and t_W = const their solution reduces to the form

$$St = 0.2\, Re_{X_1}^{-0,4} \left[1 - \left(\frac{X_0}{X}\right)^{0,9}\right]^{\frac{1}{3}}. \quad (7)$$

The same expression can also be derived from the results of Ludwieg, who related the mean heat transfer to a flat plate for $X_1 \to 0$ and t_W = const with the equation for the tangential friction stress [25]:

$$\overline{Nu}_{X_1} = 0.807 \left(\frac{X_1^2}{\mu a}\right) \tau^{\frac{1}{3}} \quad (8)$$

or

$$\overline{St} = 1.02 \left(\frac{1}{2}\bar{c}_f\right)^{1/3} Re_{X_1}^{-1/3}. \quad (9)$$

Using Eq. (4). we obtain

$$\frac{1}{2}\bar{c}_f = \frac{1}{X_1}\int_{X_0}^{X_0+X_1} 0.5 c_f(\zeta)\, d\zeta = 0.0332 \frac{Re_X}{Re_{X_1}} \left[1 - \left(\frac{X_0}{X}\right)^{0,8}\right]. \quad (10)$$

Substituting Eq. (10) into Eq. (9), we obtain

$$\overline{St}_0 = 0.338\, Re_{X_1}^{-0,4} \left[1 - \left(\frac{X_0}{X}\right)^{0,8}\right] \left[1 - \frac{X_0}{X}\right]^{-\frac{4}{15}}. \quad (11)$$

Differentiating Eq. (11), we obtain the local heat transfer equation

$$St_0 = 0.338\, Re_{X_1}^{-0,4} \left[1 - \left(\frac{X_0}{X}\right)^{0,8}\right] \left[1 - \frac{X_0}{X}\right]^{-\frac{4}{15}} \left[\frac{1}{3} + \frac{4}{15} \cdot \frac{1 - \frac{X_0}{X}}{1 - \left(\frac{X_0}{X}\right)^{0,8}}\right]. \quad (12)$$

In spite of their structural difference, Eqs. (7) and (12) give the same values for the correction factors $\varphi(X_0/X)$.

The exponent of Re_{X_1} in Eqs. (7) and (12) is $n = -0.4$. Therefore, as $X_1 \to 0$, the value of n in the heat transfer formulas for the initial section gradually decreases. In the estimates of Kestin and Richardson the change in exponent from -0.2 to -0.4 occurs in the range $Re_{X_1} = 10^4\text{–}5 \cdot 10^4$, where the inequality $10^2 < (W/\nu) \int_{X_0}^{X_1} \sqrt{0.5 c_f(\xi)}\, d\xi < 5 \cdot 10^3$ [26] holds. Ludwieg places a lower bound on the regions where the exponent approaches $n = -0.4$ by the condition $X_1^2 \tau/\mu a > 25$, which corresponds to limiting Reynolds numbers of the order of 100-200 [25].

Figure 3a. shows graphs of $\varphi(X_0/X)$ from the data of [1, 3-6, 12, 21-23, 27, 28], and also shows Eqs. (7) and (12). The value of $\varphi(X_0/X)$ deviates from unity by more than 10% only for $X_0/X > 0.75$. For $X_0/X > 0.8$ the analytical solution of [28] and the recommendations of [14] based on its conclusions give values of $\varphi(X_0/X)$ some 10-20% greater than from Eq. (6). However, it was shown in [12] that the solution of [28] is incorrect. The difference in absolute values of $\varphi(X_0/X)$ as found from Eqs. (6) and (7) and from (12) do not exceed 10-13%. Therefore, the corrections $\varphi(X_0/X)$ can be derived from the Seban formula Eq. (6) with sufficient accuracy over the whole range of Reynolds number investigated.

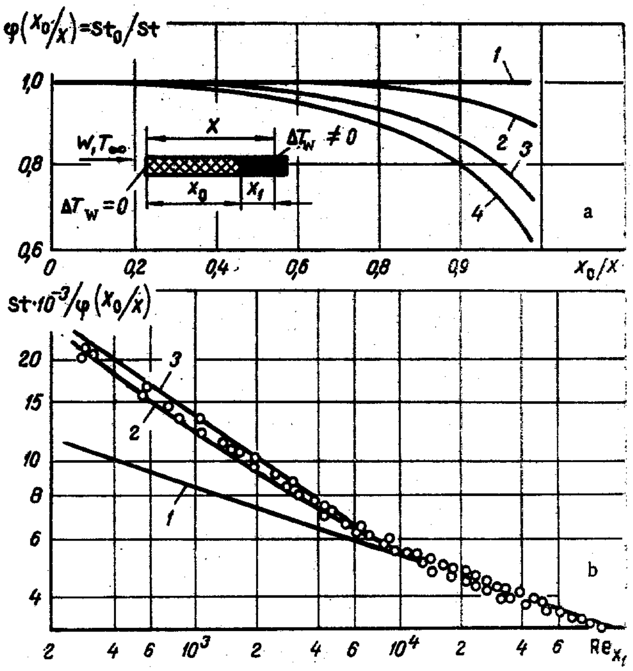

Fig. 3. Graphs of the correction $\varphi(X_0/X)$ along the initial isothermal section. a: 1) from [1, 3-6]; 2) from [14, 28]; 3) from [26] and Eq. (12); 4) from [12, 21-23, 27], and correlation of the experimental data on local heat transfer in the initial section for $X_0 > L_{dyn}$ with corrections from the Seban formula Eq. (6). b: 1) from Eq. (3); 2) Eq. (13); 3) Eq. (7).

The correction $\varphi(X_0/X)$ can be calculated from the relations given in [21, 22, 27] and the Seban formula which gives identical numerical results. But the Seban formula (6), in spite of its being somewhat laborious, has the important distinction that it gives a simpler basis for a correction to the mean heat transfer.

The results of measurements of local heat transfer coefficients on the thermal initial section of the plate, corrected using the Seban formula, are shown in Fig. 3b. The test data can be generalized by curve 2, with a scatter of ±10%, and this can be approximated to by the equation

$$St_0 = 0.018\, Re_{X_1}^{-0,16+3,25 Re_{X_1}^{-0,53}} \left[1-\left(\frac{X_0}{X}\right)^{0,9}\right]^{-\frac{1}{9}} \left[1-\frac{X_0}{X}\right]^{0,2}. \tag{13}$$

We can introduce the physical properties of the flowing medium, according to [1], in the form

$$St_0 = 0.0146\, Re_{X_1}^{-0,16+3,25 Re_{X_1}^{-0,53}}\, Pr^{-0,6} \left[1-\left(\frac{X_0}{X}\right)^{0,9}\right]^{-\frac{1}{9}} \left[1-\frac{X_0}{X}\right]^{0,2}. \tag{14}$$

Curve 2 for $Re_{X_1} > 1.5 \cdot 10^5$ coincides with curve 1, calculated by the Reynolds formula. For $Re_{X_1} > 10^4$ the analytical solution of [24] is confirmed. The region where Eq. (14) holds is bounded by the following values of the parameters: $10^2 < Re_{X_1} < 5 \cdot 10^6$; $0 < X_0/X < 0.98$; $X_0 > L_{hyd}$.

It is clear that for $X_0 = 0$ and $X_1 > L_{dyn}$ Eq. (14) describes the local heat transfer for the completely heated plate:

$$St = 0.0146\, Re_{X_1}^{-0,16+3,25 Re_{X_1}^{-0,53}}\, Pr^{-0,6}. \tag{15}$$

Mean Heat Transfer. The mean heat transfer was investigated in the heated initial section of a plate of a thickness $\delta = 8$ mm and length $L = 300$ mm. The construction of the plate has been described in [18]. The heat transfer coefficients were determined by an unsteady method [17] with t_w = const; $T_{pot}/T_w \approx 1.0$; $W = 0.5$-80.0 m/sec, $t_{pot} = 50$-55°C; $X_0 > L_{dyn}$. The basic part of the experiment was carried out using alpha-calorimeters with heated length $X_1 = 2$ mm and $X_1 = 5$ mm, in order to obtain low values of Re_{X_1}, and

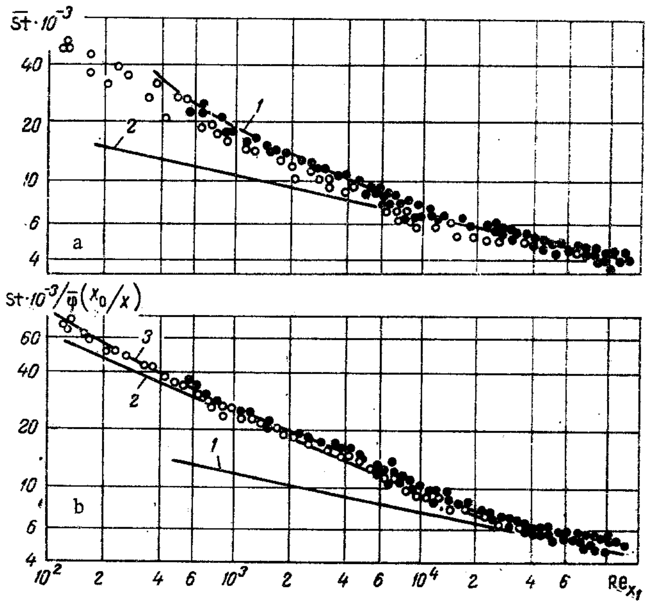

Fig. 4. Results of investigation of mean heat transfer on the heated initial section of the plate for $X_0 > L_{dyn}$ (a) and correlation of experimental data on mean heat transfer to the heated initial section for $X_0 > L_{dyn}$, with corrections according to the Seban formula (21) (b).

values of the parameter X_0/X close to unity. The results of the measurements are shown in Fig. 4a, together with the experimental data of [18]. Curve 1 represents the formula given in [18] for the mean heat transfer:

$$\overline{St} = 1.93\,Re_{X_1}^{-0.3}(\lg Re_{X_1})^{-6.1}. \tag{16}$$

Curve 2 was calculated from the equation

$$\overline{St} = 0.041\,Re_{X_1}^{-0.2}. \tag{17}$$

This formula was obtained by integration of Eq. (3b). The experimental data on mean heat transfer is qualitatively in good agreement with the results of local values of α_k (see Fig. 2a). The test points are layered in terms of the parameter X_0/X, and are grouped on the whole along curve 1, which has a slope close to $n = -0.4$ for $Re_{X_1} > 2\cdot10^4$. In the range $X_0/X = 0.7$–0.98 the separation reaches 30%, and the values of Stanton number found using the alpha-calorimeters with the heated length $X_1 = 2$ mm lie below curve 1 by 20–25%.

In analogy with Eq. (5), for the mean heat transfer coefficients we can write

$$\overline{St}_0 = \overline{St}\,\bar{\varphi}(X_0/X). \tag{18}$$

A number of recommendations for determining the correction $\bar{\varphi}(X_0/X)$ are given in [8-11, 14, 16]. These are all based on experimental data on mean heat transfer, obtained for high values of Re_{X_1}, greater than $5\cdot10^4$, as a rule.

Integrating Eq. (3) with the Seban corrections we obtain

$$\overline{St}_0 = 0.041\,Re_{X_1}^{-0.2}\cdot\left(1-\frac{X_0}{X}\right)^{-0.8}\left[1-\left(\frac{X_0}{X}\right)^{0.9}\right]^{-\frac{8}{9}}. \tag{19}$$

The relations proposed by Jakob and Dow [10], Maisel and Sherwood [11], and Tessin and Jakob [16] lead to numerical values of $\varphi(X_0/X)$ differing by no more than 10% from values calculated according to Eq. (19). The data of Edwards and Furber [8, 9] are tabulated as a function of X_0/X and Re_{X_1}. These data lie above the values calculated from Eq. (19) by 10–15%.

An analytical solution for the mean heat transfer to the heated initial section for small Re_{X_1} is given in [24]:

$$\bar{St}_0 = 0.34\,Re_{X_1}^{-0,4}\left[1-\left(\frac{X_0}{X}\right)^{0,9}\right]^{\frac{2}{3}}\left(1-\frac{X_0}{X}\right)^{\frac{2}{5}}. \qquad (20)$$

The correction $\bar{\varphi}(X_0/X)$ for $X_1 \to 0$ can also be calculated from Eq. (11), which is derived from the Ludwieg solution. In both cases the values of the correction $\bar{\varphi}(X_0/X)$ differ from those calculated using Eq. (19) by not more than 10%. Thus, as was true for a correlation of the data on local heat transfer in the heated initial section, it is convenient to use the Seban relation correction in the mean heat transfer formulas

$$\varphi(X_0/X) = \left(1-\frac{X_0}{X}\right)^{-0,8}\left[1-\left(\frac{X_0}{X}\right)^{0,9}\right]^{-\frac{8}{9}}. \qquad (21)$$

Figure 4b shows results of investigation of mean heat transfer on the heated initial section, with correction according to Eq. (19). Curve 1 corresponds to Eq. (11) and the formula of Kestin and Persen (20). The theoretical solutions are in satisfactory agreement with the experiment in the range $8\cdot10^2 < Re_{X_1} < 2.5\cdot10^4$. However, for $Re_{X_1} < 800$ a systematic deviation of the test points above curve 1 is observed. All the test data can be approximated within ±10% by curve 3, which has the equation

$$\bar{St}_0 = 0.028\,Re_{X_1}^{-0,18+2,6Re_{X_1}^{-0,4}}\left[1-\left(\frac{X_0}{X}\right)^{0,9}\right]^{-\frac{8}{9}}\left(1-\frac{X_0}{X}\right)^{-0,8}. \qquad (22)$$

For $Re_{X_1} > 10^5$ curve 3 tends to curve 2 asymptotically; curve 2 was calculated from Eq. (17) for the hypothetical case where there is complete heating of the plate and turbulence is developed from the very beginning of the flow in the boundary layer. Equation (22) is confirmed by the experiment in the range $Re_{X_1} = 2\cdot10^2 - 1.5\cdot10^5$, $X_0/X = (0-0.99)$.

NOTATION

X_1	is the heated length;
X_0	is the length of the initial unheated section;
$X = X_0 + X_1$	is the coordinate calculated from the leading edge;
δ	is the plate thickness;
L	is the plate length;
L_{dyn}	is the length of the dynamic initial section;
α_c	is the convective heat transfer coefficient;
α_r	is the radiative heat transfer coefficient;
t_w	is the plate surface temperature;
t_{pot}	is the air temperature;
W	is the air speed;
ε	is the degree of turbulence;
τ	is the tangential friction stress;
C_f	is the local friction coefficient;
E	is the emf of the heat flux gage;
k	is the heat flux gage constant;
$\varphi(X_0/X)$	is the correction to the length of the initial unheated section;
ν and η	are the coefficients of dynamic and kinematic viscosity of air;
ρ and c_p	are the density and specific heat of air;
a and λ	are the thermal diffusivity and thermal conductivity of air;
$Re_X = WX/\nu$	is the Reynolds number;
$St = \alpha_c/\rho W c_p$	is the Stanton number;
$Pr = \nu/a$	is the Prandtl number;
$Nu = \alpha_k X_1/\lambda$	is the Nusselt number.

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